

$$1. \lim_{x \rightarrow -1^-} f(x) = (-1)^2 + 2 = 3$$

$$\lim_{x \rightarrow -1^+} f(x) = (-1)^2 + 2 = 3$$

$$\lim_{x \rightarrow -1} f(x) = 2$$

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^-} f(x) \neq \lim_{x \rightarrow -1} f(x)$$

It does not exist (DNE)

$$2. (a) \begin{aligned} \lim_{x \rightarrow -2^-} f(x) &= -\infty \\ \lim_{x \rightarrow -2^+} f(x) &= \infty \end{aligned}$$

2 (a) $f(-2)$ = Does not exist (DNE)

$$(b) \lim_{x \rightarrow -2} f(x)$$

$$\lim_{x \rightarrow -2^+} f(x) = \infty$$

$$\lim_{x \rightarrow -2^-} f(x) = -\infty$$

$$\lim_{x \rightarrow -2} f(x) = \text{DNE}$$

$$(c) f(0) = 4$$

$$(d) \lim_{x \rightarrow 0} f(x)$$

$$\lim_{x \rightarrow 0^-} f(x) = -0.5$$

$$\lim_{x \rightarrow 0^+} f(x) = 4$$

$$\lim_{x \rightarrow 0} f(x) = DNE$$

$$(e) f(2) = DNE$$

$$(f) \lim_{x \rightarrow 2} f(x)$$

$$\lim_{x \rightarrow 2^-} f(x) = \frac{3}{5}$$

$$\lim_{x \rightarrow 2^+} f(x) = \frac{3}{5}$$

$$\lim_{x \rightarrow 2} f(x) = \frac{3}{5}$$

$$(g) f(4) = 0$$

$$(h) \lim_{x \rightarrow 4} f(x)$$

$$\lim_{x \rightarrow 4^-} f(x) = \infty$$

$$\lim_{x \rightarrow 4^+} f(x) = \infty$$

$$\lim_{x \rightarrow 4} f(x) = DNE$$

$$\begin{aligned}
 3. \lim_{x \rightarrow -3} (2x^2 + 5x + 4) &= 2(-3)^2 + 5(-3) + 4 \\
 &= 18 - 15 + 4 \\
 &= 3 + 4 = 7 \\
 &= 7
 \end{aligned}$$

$$\begin{aligned}
 4. \lim_{x \rightarrow 6} \cos\left(\frac{\pi x}{3}\right) &= \cos\left(\frac{\pi \cdot 6}{3}\right) = \cos(2\pi) \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 5. \lim_{x \rightarrow 0} \left(\frac{x}{x^2 - 6x} \right) &= \lim_{x \rightarrow 0} \left(\frac{1}{x - 6} \right) = \frac{1}{0 - 6} \\
 &= -\frac{1}{6}
 \end{aligned}$$

$$\begin{aligned}
 6. \lim_{x \rightarrow 1} \left(\frac{x^2 + x - 2}{x^2 - 1} \right) &= \lim_{x \rightarrow 1} \left(\frac{(x-1)(x+2)}{(x-1)(x+1)} \right) \\
 &= \lim_{x \rightarrow 1} \frac{x+2}{x+1} = \frac{1+2}{1+1} = \frac{3}{2}
 \end{aligned}$$

$$7. \lim_{x \rightarrow 0} \left(\frac{\sin x}{5x} \right) = \frac{1}{5} \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)$$

Apply L'Hopital's Rule

$$= \frac{1}{5} \lim_{x \rightarrow 0} \left(\frac{\cos(x)}{1} \right) = \frac{1}{5} \cos(0) = \frac{1}{5} = 0.2$$

$$8 \quad (a) \lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow -4^+} f(x) = 4$$

$$(b) \lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow -4^-} f(x) = 4$$

$$(c) \lim_{x \rightarrow c} f(x) = 4$$

Yes $f(-4) = 5 \neq \lim_{x \rightarrow -4} f(x)$ Not continuous

$$9. \lim_{x \rightarrow 3^+} \frac{2}{x+3} = \frac{2}{3+3} = \frac{2}{6} = \frac{1}{3}$$

$$10. f(x) = \frac{x}{x^2 + x + 8}$$

Interval = $(-\infty, \infty)$

$$11. \lim_{x \rightarrow -\frac{1}{2}^+} \frac{6x^2 + x - 1}{4x^2 - 2x - 2}$$

$$= \frac{1}{2} \lim_{x \rightarrow -\frac{1}{2}^+} \frac{6x^2 + x - 1}{2x^2 - x - 1}$$

$$= \frac{1}{2} \lim_{x \rightarrow -\frac{1}{2}^+} \frac{(2x+1)(3x-1)}{(2x+1)(x-1)}$$

$$= \frac{1}{2} \lim_{x \rightarrow -\frac{1}{2}^+} \frac{3x-1}{x-1} = \frac{1}{2} \left(\frac{3(-\frac{1}{2})-1}{-\frac{1}{2}-1} \right) = \frac{5}{6}$$

$$2. \lim_{x \rightarrow \frac{\pi}{2}^+} \left(\frac{-3}{\cos x} \right)$$

$$= -3 \lim_{x \rightarrow \frac{\pi}{2}^+} \left(\frac{1}{\cos x} \right)$$

When x approaches $\frac{\pi}{2}$ from the right, $\cos x < 0$

$$= -3 \left(\frac{-1}{0} \right) = -3(-\infty) = \infty$$

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$$f(x) = x^2 + x - 8$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = (x+h)^2 + (x+h) - 8 = x^2 + 2xh + h^2 + x + h - 8$$

$$f(x) = x^2 + x - 8$$

$$\lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + x + h - 8 - x^2 - x + 8}{h}$$

$$\lim_{h \rightarrow 0} \frac{2xh + h^2 + h}{h} = \lim_{h \rightarrow 0} (2x + h + 1)$$

$$= 2x + 0 + 1$$

$$= 2x + 1$$

$$4. \quad f(x) = 2x^2 \quad 4x - y + 2 = 0$$

$$f'(x) = 4x \quad y = 4x + 2$$

$$\text{slope, } m = 4 \quad \text{from } y = 4x + 2$$

$$f'(x) = m = 4$$

$$4 = 4x$$

$$x = 1$$

$$f(1) = 2(1)^2 = 2$$

The point is $(1, 2) = (x_1, y_1)$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 4(x - 1)$$

$$y = 4x - 4 + 2$$

$$y = 4x - 2$$

$$\text{Equation: } y = 4x - 2$$

$$5. \quad f(t) = -7t^2 + 3t + 2$$

$$f'(t) = -14t + 3$$

$$6. \quad y = 5x^4 + 8 \sin x$$

$$y' = 20x^3 + 8 \cos(x)$$

$$7. \quad g(t) = t^2 - \frac{7}{t^3} = t^2 - 7t^{-3}$$

$$g'(t) = 2t - (-3)7t^{-4} = 2t + \frac{21}{t^4}$$

$$8. f(x) = \frac{x^3 - 7x^2 + 9}{x^2}$$

$$f'(x) = \frac{x^2(3x^2 - 14x) - (x^3 - 7x^2 + 9)(2x)}{(x^2)^2}$$

$$= \frac{3x^4 - 14x^3 - 2x^4 + 14x^3 - 18x}{x^4}$$

$$= \frac{x^4 - 18x}{x^4} = \frac{x^3 - 18}{x^3}$$

$$9. f(x) = x^5 \cos x$$

$$f'(x) = x^5 (-\sin x) + (5x^4) \cos x$$

$$= -x^5 \sin x + 5x^4 \cos x$$

$$10. f(x) = \frac{1 - 5x - x^2}{x^2 - 3}$$

$$f'(x) = \frac{(x^2 - 3)(-5 - 2x) - (1 - 5x - x^2)(2x)}{(x^2 - 3)^2}$$

$$= \frac{-5x^2 - 2x^3 + 15 + 6x - 2x + 10x^2 + 2x^3}{(x^2 - 3)^2}$$

$$= \frac{5x^2 + 4x + 15}{(x^2 - 3)^2}$$